

Calculation of parity and time invariance violation in the radium atom.

V.A.Dzuba, V.V.Flambaum and J.S.M. Ginges
School of Physics, University of New South Wales, Sydney 2052, Australia
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Abstract

Parity (P) and time (T) invariance violating effects in the Ra atom are strongly enhanced due to close states of opposite parity, the large nuclear charge Z and the collective nature of P, T -odd nuclear moments. We have performed calculations of the atomic electric dipole moments (EDM) produced by the electron EDM and the nuclear magnetic quadrupole and Schiff moments. We have also calculated the effects of parity non-conservation produced by the nuclear anapole moment and the weak charge. Our results show that as a rule the values of these effects are much larger than those considered so far in other atoms (enhancement is up to 10^5 times).

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I. INTRODUCTION

The lower energy levels of radium corresponding to configurations of different parity have very close energies. This leads to a strong enhancement of the various parity (P) and time (T) invariance violating effects. In our previous paper [1] we considered the states $7s6d\ ^3D_2$ with $E = 13993.97\text{cm}^{-1}$ and $7s7p\ ^3P_1$ with $E = 13999.38\text{cm}^{-1}$, which are separated by a very small interval of $\sim 5\text{ cm}^{-1}$ ($\sim 10^{-3}\text{ eV}$). Simple estimates showed that the effects of nuclear P - and T -odd moments such as the magnetic quadrupole moment (MQM), the Schiff moment (SM) and the anapole moment (AM) are many times larger than in all atomic systems considered before. In the present paper we present more accurate calculations of these and other parity and time invariance non-conserving effects in those states of the radium atom where the effects are large. We use a relatively simple *ab initio* approximation to perform the calculations. The approximation is a reasonable compromise between the simplicity of the calculations and the accuracy of the results. It is based on relativistic Hartree-Fock (RHF) and configuration interaction (CI) methods. A minimum number of basis states are used at the CI stage of the calculations. However, important many-body effects, such as polarization of the atomic core by an external field and correlations between core and valence electrons, are included in the calculations of single-electron matrix elements. To control the accuracy of the calculations we also calculated hyperfine structure intervals and lifetimes of lower states of radium and its lighter analog barium.

Our calculations confirm the estimates done in the previous work [1] and show that the value of most P - and T -odd effects in radium is much higher than in other atoms considered before. The parity non-conserving (PNC) electric dipole transition amplitude between the ground and 3D_1 even states is about $E1_{PNC} \approx 0.8 \times 10^{-9}(Q_W/N)iea_0$, which is 100 times larger than the measured PNC amplitude in cesium [2] and about 5 times larger than the corresponding amplitude in francium [3]. The enhancement of the electron electric dipole moment (EDM) in the 3D_1 state of Ra is about 5400, which is again many times larger than corresponding values for the ground states of Fr (910) and Au (260) [4]. The transition amplitude between the ground and 3D_2 even states induced by the nuclear anapole moment is about $10^{-9}ea_0$, which is more than 10^3 times larger than a similar amplitude in Cs [2]. Also, the EDM of the Ra atom in the 3D_2 state induced by the nuclear Schiff and magnetic quadrupole moments is strongly enhanced. Both contributions (SM and MQM) are about $10^{-19}\eta\text{ e}\cdot\text{cm}$ (η is the dimensionless constant of the P -, T -odd nucleon-nucleon interaction). This is again about 10^5 times larger than the EDM of the Hg atom which currently gives the best limit on η [5]. All this makes radium a very promising candidate for the experimental study of P - and T -odd forces by means of atomic physics.

II. METHOD

We use relativistic Hartree-Fock (RHF) and configuration interaction (CI) methods to construct two-electron wave functions of the ground and lower excited states of barium and radium. The calculations start from the RHF method for a closed shell system corresponding to the ground state configuration ($6s^2$ for Ba and $7s^2$ for Ra). Since $nsnp$ and $ns(n-1)d$ configurations, with $n = 6$ for Ba and $n = 7$ for Ra, do not correspond to a closed-shell system, we calculate p and d basis states in a model HF potential. For example, to calculate

7*p* and 6*d* states of Ra, we keep all other states frozen, remove the contribution of one 7*s*-electron from the direct HF potential and use this potential to calculate the required states. The same procedure applies for Ba. Thus, we have five single-electron basis states for the CI calculations ($ns_{1/2}, np_{1/2}, np_{3/2}, (n-1)d_{3/2}, (n-1)d_{5/2}$). It turns out, however, that this simple CI approximation significantly overestimates the relative value of spin-orbit intervals for the odd-parity states and underestimates it for the even-parity states. This affects the accuracy of the calculation of *P*- and *T*-odd effects because most of them involve transitions with a change of spin which are sensitive to the value of the relativistic effects. We found that the spin-orbit intervals are sensitive to the screening of the Coulomb interaction between two external electrons (recall that Coulomb integrals contribute to the spin-orbit splitting due to the difference between the single-particle radial wave functions belonging to different components of the single-particle doublets). To improve the quality of the wave functions we introduce fitting factors f_k to the Coulomb interaction in the CI calculations (k is the multipolarity of the Coulomb interaction). It was found that multiplying all Coulomb integrals of multiplicities 0, 1 and 2 by factors $f_0 = 0.7, f_1 = 0.75, f_2 = 0.9$ significantly improves the energies and fine structure intervals of lower odd and even states of barium and radium. These factors simulate the effect of the screening of the Coulomb interaction between valence electrons and core electrons. They also compensate to some extent the effect of the incompleteness of the basis set.

To calculate values other than energy, such as the effect of electron interaction with photons and nuclear *P*- and *T*-odd fields, we also include core polarization effects (direct and exchange RPA-type corrections) and core-valence correlation effects (the Bruckner-type correlation corrections). These two effects are very important for the considered states of radium. Indeed, consider mixture of the 3D_J and $^3P_{J'}$ states by the *P*- and *T*-odd interaction W . Corresponding dominant configurations (7*s*6*d* and 7*s*7*p*) can only be mixed by a $\langle 7p|W|6d \rangle$ matrix element. However, this matrix element is extremely small in the Hartree-Fock approximation. This is because the electron interaction with *P*- and *T*-odd nuclear moments is localized in the vicinity of the nucleus where the *d*-electron does not penetrate due to the centrifugal barrier. On the other hand, the polarization of the electron core by these moments produces a long-range correction δV to the HF potential which effectively renormalizes the interaction of an external electron with the nucleus. The corresponding matrix element $\langle 7p|W + \delta V|6d \rangle$ is not small even in the case of the *p* – *d* transition due to the long-range of the renormalized interaction $W + \delta V$. Note that $\langle 7s|W|7p \rangle$ matrix elements also contribute to the mixture of the 3D_J and $^3P_{J'}$ states due to the configuration interaction. Thus, there is an interference of several factors: the *s* – *p* matrix elements are large but their contribution is suppressed due to the smallness of the configuration mixing. The *p* – *d* matrix elements are considerably smaller (although not negligible) but they appear in the dominating configurations. It cannot be said in advance which transitions are more important and as we see from our calculations there are cases when *s* – *p* transitions dominate over *p* – *d* and vice-versa (see below). The Bruckner-type correlation corrections (the correlation corrections to the single-electron wave functions) are also important, since they increase the density of an external electron on the nucleus by $\sim 30\%$ (see e.g. [6]).

The full scale inclusion of the core polarization and correlation effects into the CI calculations (see, e.g. [7]) lies beyond the framework of this research. We adopted a simplified approach in which the corresponding corrections are calculated for the single-particle matrix

elements. The relative values of the renormalization of the matrix elements by the core polarization and core-valence correlations have been extrapolated from accurate calculations of the core polarization and Bruckner-type correlation corrections for the radium positive ion. Ra^+ has a simple electronic structure - one electron above closed shells - and the corresponding procedures are well defined for it [8].

To check our method and the accuracy of our results, we calculated the hyperfine structure (hfs) constants of ^{213}Ra and ^{137}Ba . The results for the energies and hfs constants are presented in Table I. One can see that even for this very simple CI approximation the accuracy of the energies and fine structure intervals is very good. The accuracy of the hfs constants is also good for the most important states $^3\text{D}_2$ and $^3\text{P}_1$. Table II shows the effect of the core polarization (RPA) and Bruckner-type correlations (Σ) on the single-electron matrix elements. One can see that these effects play a crucial role in the $p - d$ matrix elements. However, their contribution to the $s - p$ matrix elements is also very important.

III. PARITY VIOLATION IN $7S^2 \rightarrow 7S6D$ TRANSITIONS

A. Spin-independent parity non-conservation

The Hamiltonian H_{PNC} of the interaction of an electron with the nuclear weak charge Q_W (formula (A6) in the appendix) mixes states of the same total momentum J and opposite parity. Thus, electric dipole transitions between states of initially equal parity become possible. In particular, the transition between the ground state $^1\text{S}_0$ and the excited $^3\text{D}_1$ state is enhanced due to the closeness of the opposite parity state $^1\text{P}_1$. The dominating contribution to this transition is given by

$$E1_{PNC} = \frac{\langle 7s^2 \ ^1\text{S}_0 | d_z | 7s7p \ ^3\text{P}_1 \rangle \langle 7s7p \ ^3\text{P}_1 | H_{PNC} | 7s6d \ ^3\text{D}_1 \rangle}{E(^3\text{D}_1) - E(^3\text{P}_1)}. \quad (1)$$

Apart from the enhancement, there are several suppression factors in (1). First, the electric dipole matrix element is small because of a change of spin. It is 3 to 5 times smaller than most of those amplitudes which do not change atomic spin (see Table III). Second, in the matrix element of the PNC interaction, leading configurations produce only the $p_{3/2} - d_{3/2}$ single-electron matrix element which is small. It is not zero mostly due to core polarization. However, it is about 25 times smaller than the $s_{1/2} - p_{1/2}$ matrix element. The latter contribute to the PNC amplitude due to configuration mixing. Our calculations show that the contribution of the $s - p$ transition to the PNC amplitude is about 7 times larger than the contribution of the $p - d$ transition. In spite of some suppression, the final answer is quite large:

$$^{225}\text{Ra} : E1_{PNC} = 0.77 \times 10^{-9} (Q_W/N) iea_0, \quad (2)$$

$$^{223}\text{Ra} : E1_{PNC} = 0.76 \times 10^{-9} (Q_W/N) iea_0. \quad (3)$$

This is one hundred times larger than the measured PNC amplitude in cesium [2] and about 5 times larger than the corresponding amplitude in francium [3]. Even radium isotopes have close values of the amplitudes (approximately, the effect is proportional to the number of neutrons N).

B. Anapole moment

The Hamiltonian of the electron interaction with the nuclear anapole moment is presented in the appendix (A9). Similar to the spin-independent PNC interaction, it mixes states of opposite parity and leads to non-zero E1-transition amplitudes between states of initially equal parity. However, it can also mix states with $\Delta J = 1$ and it depends on the nuclear spin, so that its contribution to transitions between different hyperfine structure components are different. The corresponding expression is very similar to (1). However, dependence on the hyperfine structure must be included (see formula (A12) in the appendix for details). This amplitude is proportional to the $\langle {}^3P_1 || \vec{\alpha}\rho(r) || {}^3D_2 \rangle$ matrix element. Contributions of different single-electron transitions into this matrix element are presented in Table IV. Note the strong cancellation between terms corresponding to $s-p$ and $p-d$ transitions. This means that an accurate inclusion of the core polarization and core-valence correlation effects is very important indeed, as has been discussed above. We believe that the fitting of the energies helps to stabilize this matrix element similar to the case of the $E1$ -transition amplitude.

The results for ${}^1S_0-{}^3D_1$ and ${}^1S_0-{}^3D_2$ transitions are presented in Table V. Note that the contribution of the anapole moment to the PNC amplitude (3) can be measured by comparing the amplitudes between different hyperfine structure components similar to what was done for cesium [2]. However, it may be much more efficient to measure the effect of the anapole moment in the ${}^1S_0-{}^3D_2$ transition because it is about ten times larger due to the small energy denominator and because the nuclear spin independent PNC interaction does not contribute to this amplitude at all due to the large change of the total electron angular momentum $\Delta J = 2$.

IV. ATOMIC ELECTRIC DIPOLE MOMENTS

A. Electron EDM

An electron electric dipole moment interacting with an atomic field mixes states with the same total momentum J and opposite parity. As a result, an atomic EDM appears. The EDM of radium in the 3D_1 state is strongly enhanced due to the closeness of the opposite parity state 3P_1 . In an approximation when only the mixture of the closest states is included, the EDM is given by

$$d = 2 \frac{\langle 7s6d {}^3D_1 | -e\mathbf{r} | 7s7p {}^3P_1 \rangle \langle 7s7p {}^3P_1 | H_{EDM} | 7s6d {}^3D_1 \rangle}{E({}^3D_1) - E({}^3P_1)}. \quad (4)$$

Calculations using formulae from the appendix give the following result

$$d = 5370d_e. \quad (5)$$

Note that a very strong enhancement is caused by the small energy denominator $E({}^3D_1) - E({}^3P_1) = 0.001292$ a.u.

B. Schiff moment

Electron interaction with the nuclear Schiff moment also produces an atomic EDM. The EDM of Ra caused by Schiff moment is strongly enhanced in the 3D_2 state. Its value is approximately given by

$$d_z = 2 \frac{\langle 7s6d \ ^3D_2 | d_z | 7s7p \ ^3P_1 \rangle \langle 7s7p \ ^3P_1 | H_{SM} | 7s6d \ ^3D_2 \rangle}{E(^3D_2) - E(^3P_1)}. \quad (6)$$

More detailed formula which include the dependence of (6) on the hyperfine structure is presented in the appendix (A19).

Table IV shows single-electron contributions to the $\langle ^3P_1 || H_{SM} || ^3D_2 \rangle$ matrix element. Note that $s - p$ transitions strongly dominate here. However, the contribution of the $p - d$ transitions is not negligible and should be included for accurate results.

Calculated values of the radium EDM induced by the Schiff moment are presented in Table VI.

C. Magnetic quadrupole moment

Electron interaction with the nuclear MQM can also produce an EDM of an atom. However, in contrast with the case of the Schiff moment, the MQM of isotopes where the nuclear spin $I < 1$ (like ^{225}Ra , where $I = 1/2$) is zero. The EDM of Ra in the 3D_2 state is given by a formula similar to (6)

$$d_z = 2 \frac{\langle 7s6d \ ^3D_2 | d_z | 7s7p \ ^3P_1 \rangle \langle 7s7p \ ^3P_1 | H_{MQM} | 7s6d \ ^3D_2 \rangle}{E(^3D_2) - E(^3P_1)}. \quad (7)$$

Again, more detailed formula can be found in the appendix (A24).

Table IV shows single-electron contributions to the $\langle ^3P_1 || A_{mk} || ^3D_2 \rangle$ matrix element. Note that in contrast to the cases of the Schiff and anapole moments, $p - d$ transitions dominate over $s - p$ transitions in this matrix element. For the anapole moment these two types of transitions contribute almost equally, while for the Schiff moment $s - p$ transitions dominate. Note that $s - p$ transitions appear due to configuration mixing only, while contribution of the $p - d$ transitions is extremely small if core polarization is not included. This indicates once more that even for a rough estimation of the time or parity invariance violating effects in Ra an inclusion of the appropriate many-body effects is essential.

Calculated values of the radium EDM induced by the magnetic quadrupole moment are presented in Table VII.

V. LIFETIMES

To plan experimental measurements of space and time invariance violation in radium it is important to know the lifetimes of the states of interest. Apart from that, comparison of the calculated and experimental lifetimes can serve as a good test of the method used for calculation of P - and T - invariance violation since the same dipole transition amplitudes

contribute in either case. As far as we know, none of the radium lifetimes have been measured so far. On the other hand, some experimental data is available for barium. Therefore, we calculated lifetimes of lower states of both atoms. Results for dipole transition amplitudes are presented in Table III and the corresponding lifetimes are in Table VIII.

For the purpose of the present work, the most important states of radium are 3P_1 , 3D_1 and 3D_2 states. The decay rate of the 3P_1 state is strongly dominated by the $^3P_1 - ^1S_0$ transition. Transitions to the 3D_1 and 3D_2 states are suppressed due to small frequencies. The $^3P_1 - ^1S_0$ dipole transition amplitude involves a change of the atomic spin and therefore is sensitive to the value of the relativistic effects. This makes the amplitude numerically unstable. This probably explains the poor agreement between different calculations (see Table VIII). However, we believe that the fitting of the fine structure which we have done for Ba and Ra (see Section II) brings the amplitude close to the correct value. This is supported by similar calculations for barium. The $^3P_1 - ^1S_0$ amplitude contributes 38% to the decay rate of the 3P_1 state of barium. Good agreement between calculated and experimental lifetimes of this state (see Table VIII) means that all transition amplitudes, including the $^3P_1 - ^1S_0$ amplitude, are calculated quite accurately.

The lifetime of the 3D_1 state of Ra is determined by the $^3D_1 - ^3P_0$ transition which is numerically stable. The lifetime of this state calculated by us is in good agreement with the estimations done by Budker and DeMille [9].

The 3D_2 state of radium is a metastable state. It decays only via electric quadrupole (E2) transition to the ground state. Calculations similar to the electric dipole transitions show that the lifetime of this state in the absence of external fields is about 15 seconds. However, measurements of the atomic EDM involve placing the atoms in a strong electric field. It is important to know how the lifetimes of the 3D_2 and 3D_1 states of Ra are affected by this field. The electric field mixes states of different parity and $\Delta J = 0, \pm 1$. If only an admixture of the nearest state is taken into account, the amplitude which determines the decay rate of a 3D_J state is given by

$$A = \frac{\langle ^1S_0 | d_z \mathcal{E} | ^3P_1 \rangle \langle ^3P_1 | d_z | ^3D_J \rangle}{E(^3P_1) - E(^3D_J)}. \quad (8)$$

Where \mathcal{E} is the electric field. This leads to the following decay rates

$$W(^3D_2) = 0.21\mathcal{E}^2 \quad (9)$$

$$W(^3D_1) = 0.25 \times 10^{-4}\mathcal{E}^2 \quad (10)$$

For an electric field of 10 kV/cm, the lifetime of the 3D_2 state is 30 μ s, while the lifetime of the 3D_1 state is 240 ms. This latter result is in good agreement with estimations done by Budker and DeMille [9]. Note that the state 3D_2 , with maximum or minimal possible projection of the total momentum on the direction of the electric field ($M = \pm 2$), cannot be mixed by this field with the 3P_1 state. Therefore its lifetime is much less affected.

VI. CONCLUSION

The radium atom turns out to be a very promising candidate for the study of time and space invariance violating effects. All such effects considered in this paper are strongly

enhanced due to the high value of the nuclear charge Z and the closeness of the opposite parity states of the atom. Moreover, the contribution of different mechanisms to the time and space invariance violating effects can be studied separately if measurements are performed for different states and different isotopes of the radium atom. For example, the atomic EDM induced by the electron EDM is strongly enhanced in the 3D_1 state, while contributions of the nuclear Schiff and magnetic quadrupole moments are strongly enhanced in the 3D_2 state. On the other hand, the magnetic quadrupole moment is zero for isotopes with nuclear spin $I = 1/2$, like ^{225}Ra , while the Schiff moment for these isotopes is not zero.

Calculations of the space and time invariance violating effects in radium reveal the importance of relativistic and many-body effects. The accuracy achieved in the present work is probably 20-30 %. However, a further improvement in accuracy is possible if such a need arises from the progress in measurements.

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APPENDIX A: WAVE FUNCTIONS AND MATRIX ELEMENTS

1. Radium wave functions

Two-electron wave functions of the ground (1S_0) and three excited (3P_1 , 3D_1 and 3D_2) states of radium used in this work for the calculation of space and time invariance violation have the following form

$$\begin{aligned}
& |7s^2 \ J = 0, L = 0, M = 0\rangle = \\
& -0.9757|7s_{\frac{1}{2},-\frac{1}{2}}7s_{\frac{1}{2},\frac{1}{2}}\rangle - 0.1150|7p_{\frac{1}{2},-\frac{1}{2}}7p_{\frac{1}{2},\frac{1}{2}}\rangle - \\
& -0.0752(|7p_{\frac{3}{2},-\frac{3}{2}}7p_{\frac{3}{2},\frac{3}{2}}\rangle - |7p_{\frac{3}{2},-\frac{1}{2}}7p_{\frac{3}{2},\frac{1}{2}}\rangle) + \\
& +0.0658(|6d_{\frac{3}{2},-\frac{3}{2}}6d_{\frac{3}{2},\frac{3}{2}}\rangle - |6d_{\frac{3}{2},-\frac{1}{2}}6d_{\frac{3}{2},\frac{1}{2}}\rangle) + \\
& +0.0702(|6d_{\frac{5}{2},-\frac{5}{2}}6d_{\frac{5}{2},\frac{5}{2}}\rangle - |6d_{\frac{5}{2},-\frac{3}{2}}6d_{\frac{5}{2},\frac{3}{2}}\rangle + |6d_{\frac{5}{2},-\frac{1}{2}}6d_{\frac{5}{2},\frac{1}{2}}\rangle),
\end{aligned} \tag{A1}$$

$$\begin{aligned}
& |7s7p \ J = 1, L = 1, M = 1\rangle = \\
& -0.9010|7s_{\frac{1}{2},\frac{1}{2}}7p_{\frac{1}{2},\frac{1}{2}}\rangle - 0.3537|7s_{\frac{1}{2},-\frac{1}{2}}7p_{\frac{3}{2},\frac{3}{2}}\rangle + 0.2042|7s_{\frac{1}{2},\frac{1}{2}}7p_{\frac{3}{2},\frac{1}{2}}\rangle - \\
& -0.0976|7p_{\frac{1}{2},-\frac{1}{2}}6d_{\frac{3}{2},\frac{3}{2}}\rangle + 0.0563|7p_{\frac{1}{2},\frac{1}{2}}6d_{\frac{3}{2},\frac{1}{2}}\rangle + 0.0512|7p_{\frac{3}{2},-\frac{1}{2}}6d_{\frac{3}{2},\frac{3}{2}}\rangle - \\
& -0.0591|7p_{\frac{3}{2},\frac{1}{2}}6d_{\frac{3}{2},\frac{1}{2}}\rangle + 0.0512|7p_{\frac{3}{2},\frac{3}{2}}6d_{\frac{3}{2},-\frac{1}{2}}\rangle - 0.0018|7p_{\frac{3}{2},-\frac{3}{2}}6d_{\frac{5}{2},\frac{5}{2}}\rangle + \\
& +0.0014|7p_{\frac{3}{2},-\frac{1}{2}}6d_{\frac{5}{2},\frac{3}{2}}\rangle - 0.0010|7p_{\frac{3}{2},\frac{1}{2}}6d_{\frac{5}{2},\frac{1}{2}}\rangle + 0.0006|7p_{\frac{3}{2},\frac{3}{2}}6d_{\frac{5}{2},-\frac{1}{2}}\rangle,
\end{aligned} \tag{A2}$$

$$\begin{aligned}
& |7s6d \ J = 1, L = 2, M = 1\rangle = \\
& -0.8660|7s_{\frac{1}{2},-\frac{1}{2}}6d_{\frac{3}{2},\frac{3}{2}}\rangle + 0.5000|7s_{\frac{1}{2},\frac{1}{2}}6d_{\frac{3}{2},\frac{1}{2}}\rangle + 0.0002|6d_{\frac{3}{2},-\frac{3}{2}}6d_{\frac{5}{2},\frac{5}{2}}\rangle - \\
& -0.0001|6d_{\frac{3}{2},-\frac{1}{2}}6d_{\frac{5}{2},\frac{3}{2}}\rangle + 0.0001|6d_{\frac{3}{2},\frac{1}{2}}6d_{\frac{5}{2},\frac{1}{2}}\rangle - 0.0001|6d_{\frac{3}{2},\frac{3}{2}}6d_{\frac{5}{2},-\frac{1}{2}}\rangle - \\
& -0.0021|7p_{\frac{1}{2},-\frac{1}{2}}7p_{\frac{3}{2},\frac{3}{2}}\rangle - 0.0012|7p_{\frac{1}{2},\frac{1}{2}}7p_{\frac{3}{2},\frac{1}{2}}\rangle,
\end{aligned} \tag{A3}$$

$$\begin{aligned}
& |7s6d \ J = 2, L = 2, M = 2\rangle = \\
& -0.8087|7s_{\frac{1}{2},\frac{1}{2}}6d_{\frac{3}{2},\frac{3}{2}}\rangle - 0.5366|7s_{\frac{1}{2},-\frac{1}{2}}6d_{\frac{5}{2},\frac{5}{2}}\rangle + 0.2400|7s_{\frac{1}{2},\frac{1}{2}}6d_{\frac{5}{2},\frac{3}{2}}\rangle - \\
& -0.0084|6d_{\frac{3}{2},\frac{1}{2}}6d_{\frac{3}{2},\frac{3}{2}}\rangle - 0.0059|6d_{\frac{3}{2},-\frac{1}{2}}6d_{\frac{5}{2},\frac{5}{2}}\rangle + 0.0053|6d_{\frac{3}{2},\frac{1}{2}}6d_{\frac{5}{2},\frac{3}{2}}\rangle - \\
& -0.0032|6d_{\frac{3}{2},\frac{3}{2}}6d_{\frac{5}{2},\frac{1}{2}}\rangle - 0.0068|6d_{\frac{5}{2},-\frac{1}{2}}6d_{\frac{5}{2},\frac{5}{2}}\rangle + 0.0091|6d_{\frac{5}{2},\frac{1}{2}}6d_{\frac{5}{2},\frac{3}{2}}\rangle +
\end{aligned} \tag{A4}$$

$$+0.0130|7p_{\frac{1}{2},\frac{1}{2}}7p_{\frac{3}{2},\frac{3}{2}}\rangle + 0.0038|7p_{\frac{3}{2},\frac{1}{2}}7p_{\frac{3}{2},\frac{3}{2}}\rangle.$$

We use the following form for the single-electron wave function

$$\psi(\mathbf{r})_{jlm} = \frac{1}{r} \begin{pmatrix} f(r)\Omega(\mathbf{r}/r)_{jlm} \\ i\alpha g(r)\tilde{\Omega}(\mathbf{r}/r)_{jlm} \end{pmatrix}. \quad (\text{A5})$$

Here $\alpha = 1/137.036$ is the fine structure constant, $\tilde{\Omega}(\mathbf{r}/r)_{jlm} = -(\vec{\sigma} \cdot \mathbf{n})\Omega(\mathbf{r}/r)_{jlm}$.

2. Spin-independent weak interaction

The Hamiltonian of the spin-independent weak interaction of an electron with the nucleus is given by [10]

$$H_{PNC} = -\frac{G}{2\sqrt{2}}\rho(r)Q_W\gamma_5, \quad (\text{A6})$$

where $G = 2.22255 \times 10^{-14}$ a.u. is the Fermi constant, ρ is the nuclear density ($\int \rho dV = 1$), $Q_W \approx -N + Z(1 - 4\sin^2\theta_W)$ is the nuclear weak charge, and γ_5 is a Pauli matrix. The matrix element of (A6) with wave functions (A5) has a form

$$\begin{aligned} \langle j_1 l_1 m_1 | H_{PNC} | j_2 l_2 m_2 \rangle &= -i\frac{G}{2\sqrt{2}}Q_W R_{PNC} \delta_{j_1 j_2} \delta_{l_1 \tilde{l}_2} \delta_{m_1 m_2}, \\ R_{PNC} &= \alpha \int \rho(f_1 g_2 - g_1 f_2) dr - \text{radial integral}, \\ \tilde{l} &= 2j - l. \end{aligned} \quad (\text{A7})$$

However, it is often more convenient to express (A8) in a form

$$\langle j_1 l_1 m_1 | H_{PNC} | j_2 l_2 m_2 \rangle = (-1)^{j_1 - m_1} \begin{pmatrix} j_1 & 0 & j_2 \\ -m_1 & 0 & m_2 \end{pmatrix} (-i)\alpha \frac{G}{2\sqrt{2}} Q_W C_{PNC} R_{PNC}, \quad (\text{A8})$$

$C_{PNC} = \sqrt{2j_1 + 1} \delta_{j_1 j_2} \delta_{l_1 \tilde{l}_2}$ - angular coefficient for the reduced matrix element.

3. Anapole moment

The Hamiltonian of the interaction of an electron with the nuclear anapole moment has the form [11]

$$H_{AM} = \frac{G}{\sqrt{2}} \frac{(\mathbf{I} \cdot \vec{\alpha})}{I(I+1)} K \kappa_a \rho(r), \quad (\text{A9})$$

where I is the nuclear spin, $K = (I + \frac{1}{2})(-1)^{I + \frac{1}{2} - l}$, l is the orbital momentum of the outermost nucleon, κ_a is a dimensionless constant proportional to the strength of the nucleon-nucleon PNC interaction [12]. The matrix elements of the Hamiltonian (A9) between the many-electron states of the atoms depend on the hyperfine structure (see, e.g. [19])

$$\begin{aligned}
\langle IJ'F|H_{AM}|IJF\rangle &= \frac{G}{\sqrt{2}} \frac{K\kappa_a}{I(I+1)} (-1)^{F+I+J'} \begin{Bmatrix} I & I & 1 \\ J & J' & F \end{Bmatrix} \times \\
&\times \sqrt{I(I+1)(2I+1)} \langle J' || \vec{\alpha}\rho(r) || J \rangle, \\
\mathbf{F} &= \mathbf{I} + \mathbf{J}, \mathbf{J} - \text{atomic total momentum.}
\end{aligned} \tag{A10}$$

The electron part of the operator (A9) is $\vec{\alpha}\rho(r)$. Its single-electron matrix elements over states (A5) have a form

$$\begin{aligned}
\langle j_1 l_1 m_1 | \vec{\alpha}\rho(r) | j_2 l_2 m_2 \rangle &= (-1)^{j_1 - m_1} \begin{pmatrix} j_1 & 1 & j_2 \\ -m_1 & q & m_2 \end{pmatrix} (C_{1AM} R_{1AM} + C_{2AM} R_{2AM}), \\
C_{1AM} &= (-1)^{j_1 + l_2 + \frac{1}{2}} \sqrt{6(2j_1 + 1)(2j_2 + 1)} \begin{Bmatrix} \frac{1}{2} & j_1 & l_2 \\ j_2 & \frac{1}{2} & 1 \end{Bmatrix} \delta_{l_1 l_2}, \\
C_{2AM} &= (-1)^{j_1 + l_1 + \frac{3}{2}} \sqrt{6(2j_1 + 1)(2j_2 + 1)} \begin{Bmatrix} \frac{1}{2} & j_1 & l_1 \\ j_2 & \frac{1}{2} & 1 \end{Bmatrix} \delta_{l_1 \tilde{l}_2}, \\
R_{1AM} &= -4\pi\alpha \int g_1 f_2 dr, \\
R_{2AM} &= -4\pi\alpha \int f_1 g_2 dr.
\end{aligned} \tag{A11}$$

The dominating contribution to the z -component of the parity non-conserving electric dipole transition amplitude between the 1S_0 and 3D_1 states of Ra induced by the anapole moment is given by

$$\begin{aligned}
E1_{PV} &= (-1)^{F-f} \begin{pmatrix} F & 1 & F' \\ -f & 0 & f \end{pmatrix} (-1)^{4F'+J+J'+2I+1} \frac{G}{\sqrt{2}} K\kappa_a \sqrt{\frac{2I+1}{I(I+1)}} \times \\
&\times \sqrt{(2F+1)(2F'+1)} \begin{Bmatrix} J' & I & F' \\ F & 1 & J \end{Bmatrix} \begin{Bmatrix} I & I & 1 \\ J & J' & F' \end{Bmatrix} \frac{\langle 7s^2 || E1 || 7s7p \rangle \langle 7s7p || \vec{\alpha}\rho(r) || 7s6d \rangle}{E_{7s6d} - E_{7s7p}}.
\end{aligned} \tag{A12}$$

Here $\mathbf{F} = \mathbf{I} + \mathbf{J}$, $f = \min(F, F')$.

4. Electron EDM

The Hamiltonian of the interaction of the **electron EDM** d_e with the atomic electric field \mathbf{E} has the form [10]

$$H_{EDM} = -d_e \beta (\boldsymbol{\Sigma} \cdot \mathbf{E}), \tag{A13}$$

where

$$\beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \boldsymbol{\Sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}, \quad \mathbf{E} = -\nabla V(\mathbf{r}).$$

The atomic EDM induced by (A13) can be calculated as an average value of the operator of the dipole moment over states mixed by an operator similar to (A13)

$$H'_{EDM} = -d_e (\beta - 1) (\boldsymbol{\Sigma} \cdot \mathbf{E}). \tag{A14}$$

Its single-electron matrix elements have a form

$$\begin{aligned}
\langle j_1 l_1 m_1 | H'_{EDM} | j_2 l_2 m_2 \rangle &= (-1)^{j_1 - m_1} \begin{pmatrix} j_1 & 0 & j_2 \\ -m_1 & 0 & m_2 \end{pmatrix} d_e C_{EDM} R_{EDM}, \\
C_{EDM} &= \sqrt{2j_1 + 1} \delta_{j_1 j_2} \delta_{l_1 l_2}, \\
R_{EDM} &= 2\alpha^2 \int g_1 \frac{dV}{dr} g_2 dr.
\end{aligned} \tag{A15}$$

Note that the selection rules and the angular coefficients are the same as for the spin independent weak interaction (A6), while the radial integrals are different.

5. Schiff moment

The Hamiltonian of the interaction of an electron with the nuclear Schiff moment has the form [13]

$$H_{SM} = 4\pi \mathbf{S} \cdot \nabla \rho(r), \tag{A16}$$

$\mathbf{S} = S\mathbf{I}/I$, S is Schiff moment. Many-electron matrix elements of (A16) depend on the hyperfine structure similar to (A9)

$$\langle I J' F | H_{SM} | I J F \rangle = (-1)^{F+I+J'} \begin{Bmatrix} I & I & 1 \\ J & J' & F \end{Bmatrix} S \sqrt{\frac{I(I+1)(2I+1)}{I}} \langle J' || 4\pi \nabla \rho(r) || J \rangle. \tag{A17}$$

The electron part of the operator (A16) is $4\pi \nabla \rho(r)$. Its single-electron matrix elements over states (A5) have the form

$$\begin{aligned}
\langle j_1 l_1 m_1 | 4\pi \nabla \rho(r) | j_2 l_2 m_2 \rangle &= (-1)^{j_1 - m_1} \begin{pmatrix} j_1 & 1 & j_2 \\ -m_1 & q & m_2 \end{pmatrix} C_{SM} R_{SM}, \\
C_{SM} &= (-1)^{j_2 + \frac{3}{2}} \sqrt{(2j_1 + 1)(2j_2 + 1)} \begin{pmatrix} j_1 & j_2 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix} \xi(l_1 + l_2 + 1), \\
\xi(x) &= \begin{cases} 1, & \text{if } x \text{ is even} \\ 0, & \text{if } x \text{ is odd} \end{cases}, \\
R_{SM} &= -4\pi \int (f_1 f_2 + \alpha^2 g_1 g_2) \frac{d\rho}{dr} dr.
\end{aligned} \tag{A18}$$

The EDM of Ra induced by the nuclear Schiff moment for a particular hyperfine structure component of the 3D_2 state is approximately given by

$$\begin{aligned}
d_z &= 2 \begin{pmatrix} F & 1 & F \\ -F & 0 & F \end{pmatrix} (-1)^{2F+2I+J+J'} \begin{Bmatrix} J' & I & F \\ F & 1 & J \end{Bmatrix} \begin{Bmatrix} I & I & 1 \\ J & J' & F \end{Bmatrix} \sqrt{\frac{I(I+1)(2I+1)}{I}} \times \\
&\quad (2F+1) S \frac{\langle 7s6d \ ^3D_J || E1 || 7s7p \ ^3P_{J'} \rangle \langle 7s7p \ ^3P_{J'} || 4\pi \nabla \rho(r) || 7s6d \ ^3D_J \rangle}{E_{7s6d} - E_{7s7p}}.
\end{aligned} \tag{A19}$$

6. Magnetic quadrupole moment

The Hamiltonian of the interaction of an electron with the nuclear magnetic quadrupole moment has the form [13]

$$\begin{aligned}
H_{MQM} &= -\frac{M}{4I(2I-1)}t_{mk}A_{mk}, \\
t_{mk} &= I_m I_k + I_k I_m - \frac{2}{3}\delta_{km}I(I+1), \\
A_{mk} &= \epsilon_{nim}\alpha_n\partial_i\partial_k\frac{1}{r}.
\end{aligned} \tag{A20}$$

Its many-electron matrix element is

$$\langle IJ'F|H_{MQM}|IJF\rangle = \frac{3M}{8I(2I-1)}\sqrt{5(2F+1)(2I+3)(I+1)(2I+1)I(2I-1)} \times \left\{ \begin{matrix} 2 & 2 & 0 \\ J' & I & F \\ J & I & F \end{matrix} \right\} \langle J'||A_{mk}||J\rangle. \tag{A21}$$

$$\tag{A22}$$

The single-electron matrix elements of the operator A_{mk} have the form

$$\langle j_1 l_1 m_1 | A_{mk} | j_2 l_2 m_2 \rangle = (-1)^{j_1 - m_1} \begin{pmatrix} j_1 & 2 & j_2 \\ -m_1 & q & m_2 \end{pmatrix} (C_{1MQM} + C_{2MQM}) R_{MQM}, \tag{A23}$$

$$C_{1MQM} = (-1)^{j_2 - \frac{1}{2}} \frac{4}{3} \sqrt{(2j_1+1)(2j_2+1)} \begin{pmatrix} j_1 & j_2 & 2 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix} \xi(l_1 + l_2 + 1),$$

$$C_{2MQM} = (-1)^{j_1 + j_2 + l_2 + 1} 4 \sqrt{5(2j_1+1)(2j_2+1)(2l_1+1)(2l_2+1)} \times$$

$$\begin{pmatrix} l_1 & 1 & l_2 \\ 0 & 0 & 0 \end{pmatrix} \left\{ \begin{matrix} 1 & l_1 & l_2 \\ 2 & j_1 & j_2 \\ 1 & \frac{1}{2} & \frac{1}{2} \end{matrix} \right\},$$

$$R_{MQM} = \alpha \int F(r)(g_1 f_2 + f_1 g_2) dr,$$

$$\text{where } F(r) = \begin{cases} r/r_N^4, & \text{if } r \leq r_N \\ 1/r^3, & \text{if } r > r_N \end{cases},$$

r_N – nuclear radius.

The EDM of Ra induced by the nuclear MQM for a particular hyperfine structure component of the 3D_J state is approximately given by

$$\begin{aligned}
d_z &= 2 \begin{pmatrix} F & 1 & F \\ -F & 0 & F \end{pmatrix} (-1)^{F+I+J} (2F+1)^{\frac{3}{2}} \frac{3\sqrt{5}}{4} M \sqrt{\frac{(2I+3)(I+1)(2I+1)}{I(2I-1)}} \times \\
&\left\{ \begin{matrix} J' & I & F \\ F & 1 & J \end{matrix} \right\} \left\{ \begin{matrix} 2 & 2 & 0 \\ J & I & F \\ J' & I & F \end{matrix} \right\} \frac{\langle 7s6d \ ^3D_J || E1 || 7s7p \ ^3P_{J'} \rangle \langle 7s7p \ ^3P_{J'} || A_{mk} || 7s6d \ ^3D_J \rangle}{E_{7s6d} - E_{7s7p}}. \tag{A24}
\end{aligned}$$

For the case of the EDM in the 3D_J state, $J' = 1, J = 2$ in (A24).

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TABLES

TABLE I. Energies and hyperfine structure constants of lower excited states of ^{137}Ba ($I=3/2$, $\mu=0.937365$) and ^{213}Ra ($I=1/2, \mu=0.6133$).

Atom	State	Energies (cm^{-1})		hfs constant A (MHz)	
		Calc.	Exper. [14]	Calc.	Exper. [15]
Ba	$6s5d$ $^3\text{D}_1$	9225	9034	-632	-520.5
	$^3\text{D}_2$	9346	9216	357	415.9
	$^3\text{D}_3$	9554	9596	504	456.6
	$^1\text{D}_2$	12147	11395	-26	-82.18
	$6s6p$ $^3\text{P}_0$	12203	12226		
	$^3\text{P}_1$	12577	12637	1233	1150.59
	$^3\text{P}_2$	12464	13514	878	
	$^1\text{P}_1$	18042	18060	-48	-109.2
Ra	$7s7p$ $^3\text{P}_0$	12971	13078		
	$^3\text{P}_1$	13926	13999	8058	
	$^3\text{P}_2$	16660	16689	4637	
	$^1\text{P}_1$	21033	20716	-1648	-2315
	$7s6d$ $^3\text{D}_1$	13893	13716	-4108	
	$^3\text{D}_2$	14042	13994	1749	
	$^3\text{D}_3$	14299	14707	2744	
	$^1\text{D}_2$	17750	17081	-320	

TABLE II. Single-electron matrix elements of the P - and T -odd interactions for radium (presented reduced matrix elements of an electron part of the Hamiltonian as specified in the table, see Appendix for details). All values are in atomic units.

Matrix element	Approximation		Even or odd ^c
	RHF ^a	RHF+RPA+ Σ^b	
Spin-independent PNC interaction, $H = \rho(r)\gamma_5$			
$\langle 7s_{1/2} H 7p_{1/2} \rangle$	-2769	-3832	Odd
$\langle 7p_{3/2} H 6d_{3/2} \rangle$	0.004	-146.8	Odd
Nuclear Anapole moment, $H = \vec{\alpha}\rho(r)$			
$\langle 7s_{1/2} H 7p_{1/2} \rangle$	-503	-577	Odd
$\langle 7s_{1/2} H 7p_{3/2} \rangle$	-0.508	20.26	Even
$\langle 7p_{1/2} H 6d_{3/2} \rangle$	-0.024	-66.29	Even
$\langle 7p_{3/2} H 6d_{3/2} \rangle$	0.0006	-29.99	Odd
$\langle 7p_{3/2} H 6d_{5/2} \rangle$	0	11.71	Even
Electron dipole moment, $H = (\beta - 1)\Sigma\mathbf{E}$			
$\langle 7s_{1/2} H 7p_{1/2} \rangle$	12.06	17.05	Even
$\langle 7p_{3/2} H 6d_{3/2} \rangle$	0.556	2.082	Even
Nuclear Schiff moment, $H = 4\pi\nabla\rho(r)$			
$\langle 7s_{1/2} H 7p_{1/2} \rangle$	-44400	-63027	Even
$\langle 7s_{1/2} H 7p_{3/2} \rangle$	-32550	-56730	Odd
$\langle 7p_{1/2} H 6d_{3/2} \rangle$	-1497	1873	Odd
$\langle 7p_{3/2} H 6d_{3/2} \rangle$	-0.03	2767	Even
$\langle 7p_{3/2} H 6d_{5/2} \rangle$	-0.07	8163	Even
Nuclear Magnetic quadrupole moment, $H = A_{mk}$			
$\langle 7s_{1/2} H 7p_{3/2} \rangle$	17.28	25.06	Odd
$\langle 7p_{1/2} H 6d_{3/2} \rangle$	2.831	2.933	Odd
$\langle 7p_{1/2} H 6d_{5/2} \rangle$	-0.2017	6.631	Even
$\langle 7p_{3/2} H 6d_{5/2} \rangle$	0.5389	4.011	Odd

^aRelativistic Hartree-Fock

^bCore polarization and core-valence correlation interaction are included

^cEven means that $\langle i || H || j \rangle = \langle j || H || i \rangle$; odd means that $\langle i || H || j \rangle = -\langle j || H || i \rangle$.

TABLE III. E1-transition amplitudes for Ba and Ra ($|\langle i||d_z||j\rangle|a_0$).

Transition		Ba		Ra	
i	j	Amplitude	Frequency $\epsilon_i - \epsilon_j$ (a.u.)	Amplitude	Frequency $\epsilon_i - \epsilon_j$ (a.u.)
3P_0	3D_1	2.3121	0.01473	3.0449	-0.002904
3P_1	1S_0	0.4537	0.05758	1.0337	0.06379
3P_1	3D_1	2.0108	0.01641	2.6389	0.001292
3P_1	3D_2	3.4425	0.01559	4.4399	0.0000247
3P_1	1D_2	0.1610	0.00566	0.0467	-0.01404
3P_2	3D_1	0.5275	0.02042	0.7166	0.01354
3P_2	3D_2	2.024	0.01959	2.7283	0.01228
3P_2	3D_3	4.777	0.01785	6.3728	0.009027
3P_2	1D_2	0.1573	0.00966	0.1499	-0.001790
1P_1	1S_0	5.236	0.08229	5.4797	0.09439
1P_1	3D_1	0.1047	0.04113	0.4441	0.03189
1P_1	3D_2	0.4827	0.04030	1.188	0.03063
1P_1	1D_2	1.047	0.03037	2.4053	0.01656

TABLE IV. Single-electron contributions to the two-electron matrix element $\langle ^3P_1||H||^3D_2\rangle$. Dash means no contribution due to selection rules. Zero means very small contribution. Same units as in Table II.

Transition	$H = -e\mathbf{r}^a$	$H = \alpha\rho(r)^b$	$H = 4\pi\nabla\rho(r)^c$	$H = A_{mk}^d$
$7s_{1/2} - 7p_{1/2}$	-0.3677	58.78	6421	-
$7p_{1/2} - 7s_{1/2}$	0.0215	3.431	-375	-
$7s_{1/2} - 7p_{3/2}$	-0.1306	-0.515	1441	-1.565
$7p_{3/2} - 7s_{1/2}$	-0.0585	0.230	645	-0.289
$7p_{1/2} - 6d_{3/2}$	0.0020	0.029	-1	-0.003
$6d_{3/2} - 7p_{1/2}$	3.856	-54.08	-1528	-1.848
$7p_{1/2} - 6d_{5/2}$	-	-	-	0
$6d_{5/2} - 7p_{1/2}$	-	-	-	-2.032
$7p_{3/2} - 6d_{3/2}$	0.0004	0.006	-1	-
$6d_{3/2} - 7p_{3/2}$	-0.2470	3.522	325	-
$7p_{3/2} - 6d_{5/2}$	0	0	0	0
$6d_{5/2} - 7p_{3/2}$	1.364	2.442	-1702	-0.736
Total	4.4399	13.85	5226	-6.473

^aFor $E1$ transition amplitude^bFor anapole moment contribution^cFor Schiff moment contribution^dFor Magnetic quadrupole moment contribution

TABLE V. Parity non-conserving E1-transition amplitude induced by nuclear anapole moment

I	F	F'	$\langle d_z \rangle$ in units $10^{-10} \kappa_a i e a_0$	
			$^1S_0-^3D_1$	$^1S_0-^3D_2$
0.5	0.5	1.5	2.05	-20.3
1.5	1.5	0.5	-0.58	5.7
	1.5	1.5	-1.4	13.8
	1.5	2.5	1.3	-12.9

 TABLE VI. EDM of Ra atom in the 3D_2 state induced by nuclear Schiff moment

I	F	d_z (a.u.)		d_z (ecm)
0.5	1.5	$-0.94 \times 10^8 S$	$-0.19 \times 10^{-11} \eta^a$	$-0.36 \times 10^{-19} \eta$
1.5	0.5	$-0.16 \times 10^8 S$	$-0.42 \times 10^{-11} \eta^b$	$-0.80 \times 10^{-19} \eta$
1.5	1.5	$-0.30 \times 10^9 S$	$-0.81 \times 10^{-11} \eta^b$	$-0.15 \times 10^{-18} \eta$
1.5	2.5	$-0.28 \times 10^9 S$	$-0.76 \times 10^{-11} \eta^b$	$-0.14 \times 10^{-18} \eta$

^aNuclear Schiff moment S is assumed to be $S = 400 \times 10^8 \eta \text{ e fm}^3$ [20]

^b $S = 300 \times 10^8 \eta \text{ e fm}^3$ [20]

 TABLE VII. EDM of ^{223}Ra isotope ($I = 3/2$) in the 3D_2 state induced by nuclear magnetic quadrupole moment

F	d_z^a	d_z^b
0.5	$1344 M m_e$	$7.4 \times 10^{-20} \eta \text{ e} \cdot \text{cm}$
1.5	$1292 M m_e$	$7.0 \times 10^{-20} \eta \text{ e} \cdot \text{cm}$
2.5	$-806 M m_e$	$-4.4 \times 10^{-20} \eta \text{ e} \cdot \text{cm}$

^aIn terms of nuclear magnetic quadrupole moment M

^b M is assumed to be $M = 10^{-19} (\eta/m_p) \text{ e} \cdot \text{cm}$, [21] where m_p is the proton mass

TABLE VIII. Lifetimes of lower short-living states of Ba and Ra

Atom	State	Lower states to decay to via E1-transitions	Lifetime	
			This work	Other data
Ba	3P_0	3D_1	$2.83 \mu s$	
	3P_1	$^1S_0, ^3D_1, ^3D_2, ^1D_2$	$1.37 \mu s$	$1.2 \mu s$ ^a
	3P_2	$^3D_1, ^3D_2, ^3D_3, ^1D_2$	$1.41 \mu s$	
	1P_1	$^1S_0, ^3D_1, ^3D_2, ^1D_2$	$9.1 ns$	$8.5 ns$ ^a
Ra	3P_1	$^1S_0, ^3D_1, ^3D_2$	$505 ns$	$420 ns$ ^b , $250 ns$ ^c
	3P_2	$^3D_1, ^3D_2, ^3D_3$	$74.6 ns$	
	3D_1	3P_0	$617 \mu s$	$800 \mu s$ ^d
	1D_2	$^3P_1, ^3P_2$	$38 ms$	
	1P_1	$^1S_0, ^3D_1, ^3D_2, ^1D_2$	$5.5 ns$	

^aReference [16]

^bReference [17]

^cReference [18]

^dEstimation, Reference [9]